Comparison of Binomial and Gaussian Distributions for Evaluation of Optical DS-CDMA System BER Performance

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Outlines

• Asynchronous optical DS-CDMA system

• **Compare** Binomial and Gaussian distributions

• Conditions required to approximate Binomial distribution by Gaussian

• **Analysis**

• **Conclusions**
DS-CDMA System

- Uses spreading sequences as message separator, i.e. \((n,w,1,1)\) OOC

where

\(n\) and \(w\) are the sequence length and the code weight, respectively,

1,1 are the auto- and cross- correlation constraints

Example, \((5,2,1,1)\) OOC

\[ n=5 \]

\[ w=2 \]
DS-CDMA System - contd.

Auto correlation

Sequence 1

Sequence 1

Correlator o/p

1

2

Sequence 1

Sequence 2

Correlator o/p

1

Cross-correlation
DS-CDMA System – contd.

- Can be categorised into:
  - Synchronous
  - **Asynchronous** (with chip being synchronised)
  - Fully asynchronous
Asynchronous DS-CDMA

- Transmitter and receiver are synchronised at chip level, but spreading sequence phase is random

\[ T_c = \text{chip rate} \]
Asynchronous DS-CDMA

• There are $F$ multiple simultaneous links with a set of $F$ spreading code $(n,w,1,1)$ OOC separating them

\[ n = F \cdot w \cdot (w-1) + 1 \]

• Each receiver signal will be affected by the signals from the other $F-1$ signals, known as **Multiple-Access-Interference (MAI)**.

• The probability of interference from one source is:

\[ p = \frac{w^2}{2n} \]
MAI – Binomial Distribution

• Assuming: \( P(1) = P(0) \), the probability of error due to the total interference from the \((F-1)\) is:

\[
P_{E|B} = \frac{1}{2} \cdot \sum_{i=Th}^{F-1} \binom{F-1}{i} p^i q^{F-1-i}
\]

• Is a discrete distribution based on two outcomes \( A \) and \( \overline{A} \) with probability and

\[
P(A) = p \quad P(\overline{A}) = 1 - p = q
\]

• The probability that the outcome \( A \) to occur (in any order) \( k \) times or more in \( N \) trials is obtained from:

\[
P_{E|B} = \sum_{l=k}^{N} \binom{N}{l} p^k q^{N-l}
\]
MAI – Gaussian Distribution

- Thus the probability of error is:

\[
P_{E|G} = \frac{1}{2} \cdot \frac{1}{\sigma \sqrt{2\pi}} \int_{i=Th}^{\infty} \exp \left[ -\frac{1}{2} \left( \frac{i-\mu}{\sigma} \right)^2 \right] di
\]

With mean \( \mu = (F-1)p \) and variance \( \sigma^2 = (F-1)pq \)

- Is continuous and can approximate for Binomial using the function:

\[
P_{E|G} = \frac{1}{\sigma \sqrt{2\pi}} \int_{l=k}^{\infty} \exp \left[ -\frac{1}{2} \left( \frac{l-\mu}{\sigma} \right)^2 \right] dl
\]

With mean \( \mu = Np \), variance \( \sigma^2 = Npq \) and \( N = \text{large} \)
Gaussian Distribution

• For GD to closely approximate BD two conditions must be satisfied:

\[ \mu - 2\sigma = Np - 2\sqrt{Npq} \]
\[ \mu + 2\sigma = Np + 2\sqrt{Npq} \]

Where \( N \) is the number of trials

• Or

\[ Np \geq 4 \quad \text{and} \quad Nq \geq 4 \]
Gaussian Distribution - contd

$p=0.6$ and $q=0.4$
Gaussian Distribution – contd.
Probability Distribution Function

\[(n, 8, 1, 1) \text{ OOC} \times F = 10\]
\[\text{o } F = 1000\]

Note: \(q > P\)
Probability of Error for \((n,8,1,1)\)

\(\text{OOC}\)

\[ \times F = 10 \]

\(\circ F = 1000 \)

Binomial

Gaussian

\[ P_E \]

Variable / Threshold
Is DS-CDMA Really Gaussian? Analysis

• For GD ~ BD, \( Np > 4 \) and \( Nq > 4 \)
  and therefore
  \[
  (F-1)p > 4 \quad \text{and} \quad (F-1)q > 4.
  \]

• Substituting for \( p = \frac{w^2}{2n} \) and \( n = F \cdot w \cdot (w-1)+1 \) results in:

\[
(F - 1)p = (F - 1)\frac{w^2}{2n} = \frac{(F - 1)w^2}{2[Fw(w-1)+1]}
\]

\[
(F - 1)q = (F - 1)\left(1 - \frac{w^2}{2n}\right) = (F - 1) - \frac{(F - 1)w^2}{2[Fw(w-1)+1]}
\]
Analysis – contd.

As $F \to \infty$

$$(F - 1)p = \frac{(F - 1)w^2}{2[Fw(w-1)+1]} \to \frac{w}{2(w-1)}$$

$$(F - 1)q = (F - 1) - \frac{(F - 1)w^2}{2[Fw(w-1)+1]} \to (F - 1) - \frac{w}{2(w-1)}$$
## Results for \((F-1)p\) only

<table>
<thead>
<tr>
<th>(w)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>8</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u/(F-1)p)</td>
<td>1</td>
<td>0.75</td>
<td>0.667</td>
<td>0.571</td>
<td>0.526</td>
</tr>
</tbody>
</table>
Conclusions

• We have shown that the GD is a close approximation of BD for small value of $w$, for Asy. DS-CDMA.

• As $F$ increases only one of the conditions for GD~BD is satisfied.
• For a given value of $F$, and a large values of $w$ we have shown that $P_E$ for both distributions diverge rather than converge.

• No matter how large MAI, the $(F-1)$, the MAI distribution, which is Binominal, remain unchanged and is not close to Gaussian distribution.